Resistance Coverage of Memory Polarized Mho Distance Elements

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Arcing Faults

- AR. Van C. Warrington, "*Reactance Relays Negligibly Affected by Arc Impedance*", Electrical World, September, 1931, pp. 502-505, 1931.
- Published results are from field tests done HV systems of New England and Tennessee Electric Power Company (150-1000A)
- The voltage across the arc is in phase with the current – It is Resistive

Arc Resistance

- Empirical formulae used in the industry
  - Warrington's Formula
  - Rarc = $\frac{8750\,L}{I^{1.4}}$ L –Length of the arc and I- current through the arc.
  - Voltage drop across the arc, $V_{arc} = \frac{8750\,L}{I^{0.4}}$
  - Voltage Decreases with the increase in current
  - Rarc = $\frac{8750\,L(1+3Ut)}{I^{1.4}}$

U: Cross wind speed, mph, t:Time in seconds

Westinghouse Experiments

- A.P. Strom, "*Long 60 Cycle Arc in Air*", Transactions of AIEE, Vol. 65, PP 113-117, 1946.
- Used 1/8 – 48 inch arc lengths with currents from 68A- 22kA
- Average Voltage Gradient: 31-38V/inch
- Arc voltage varies considerable from one cycle to another (21.5 V-50V/in with an average of 34V/in or 408V/ft)

Arc Resistance

- Empirical formula used in the industry (USA)
  - Formula suggested in Westinghouse Relay book
  - Rarc = $\frac{440\ L}{I}$,L –Length of the arc and I- current through the arc.
  - Voltage drop across the arc, V_{arc} = 440L
  - Voltage drop is dependent only on the length of the arc.

More Recent Research

- Prof. Terzija's research – from 2000 onwards
- According to Prof. Vladimir Terzija -majority of faults (over 90%) are arcing faults.
- Questioned accuracy of Warrington's formula.
- Test Current range 2kA-12kA
- Arc Length 0.17m -2m (0.56ft-6.56ft)
- Rectangular Voltage waveforms with THD of 30%
- Two approaches to derive Equation for voltage Gradient –$I * R = \frac{2\sqrt{2}\, E_a L}{\pi}$ from Arc modeling
  $=$ (A+B/I)*L from spectrum domain analysis
- E_a (1200 to 1500V/m or 365V - 457V/ft)
- A= 855.3 (260.7 if L is in ft) B=4501.6 (1372 if L is in ft)

Voltage Across the arc

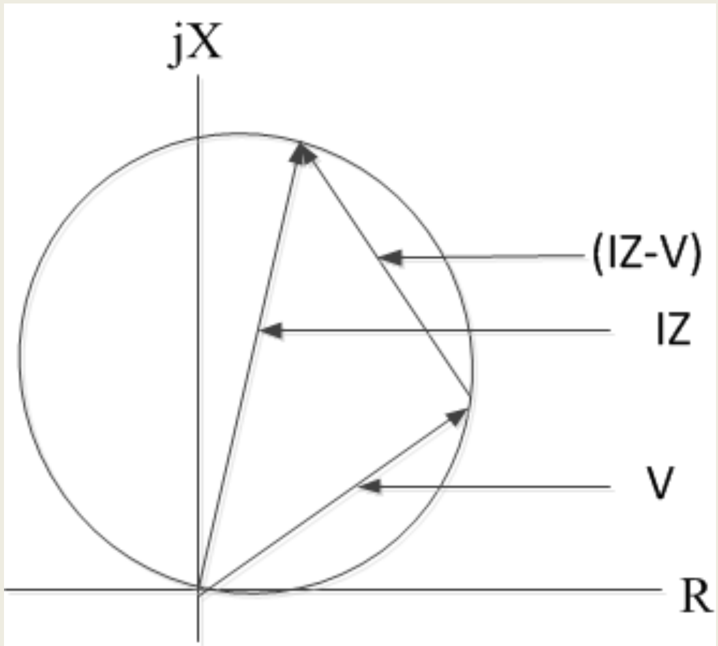
- Voltage drop across the arc either decreases with infeed or remains constant. It is proportional to the length of the arc.

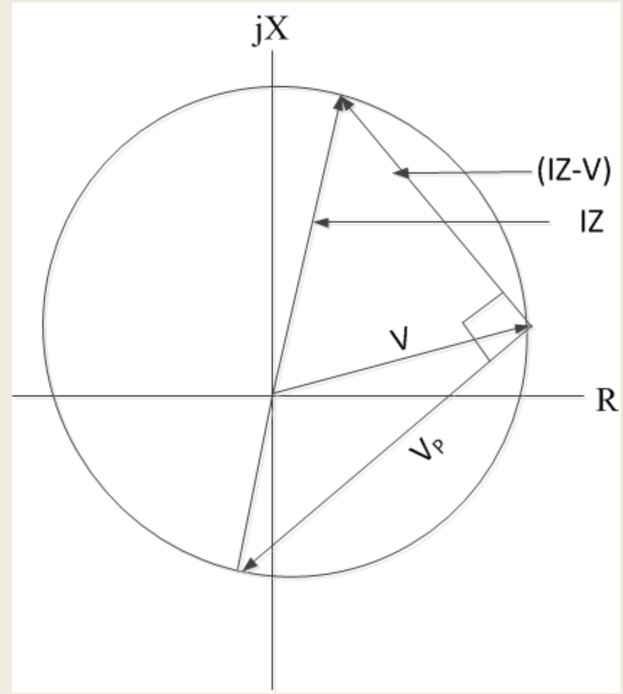
- Voltage waveform has THD of 30% (Distorted Waveform)

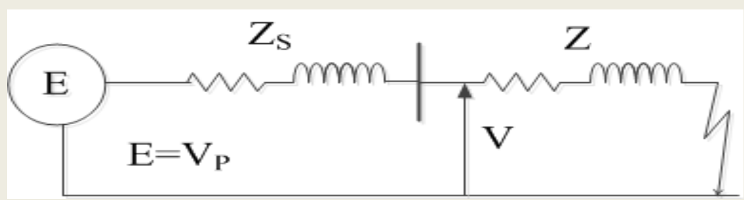
Mho Distance element

- Compensated voltage(IZ-V), where I and V are voltage and current at the relay location
- Polarizing voltage, V_P – This could be the fault voltage or pre-fault or Voltage from healthy phases(s).

Mho Characteristics

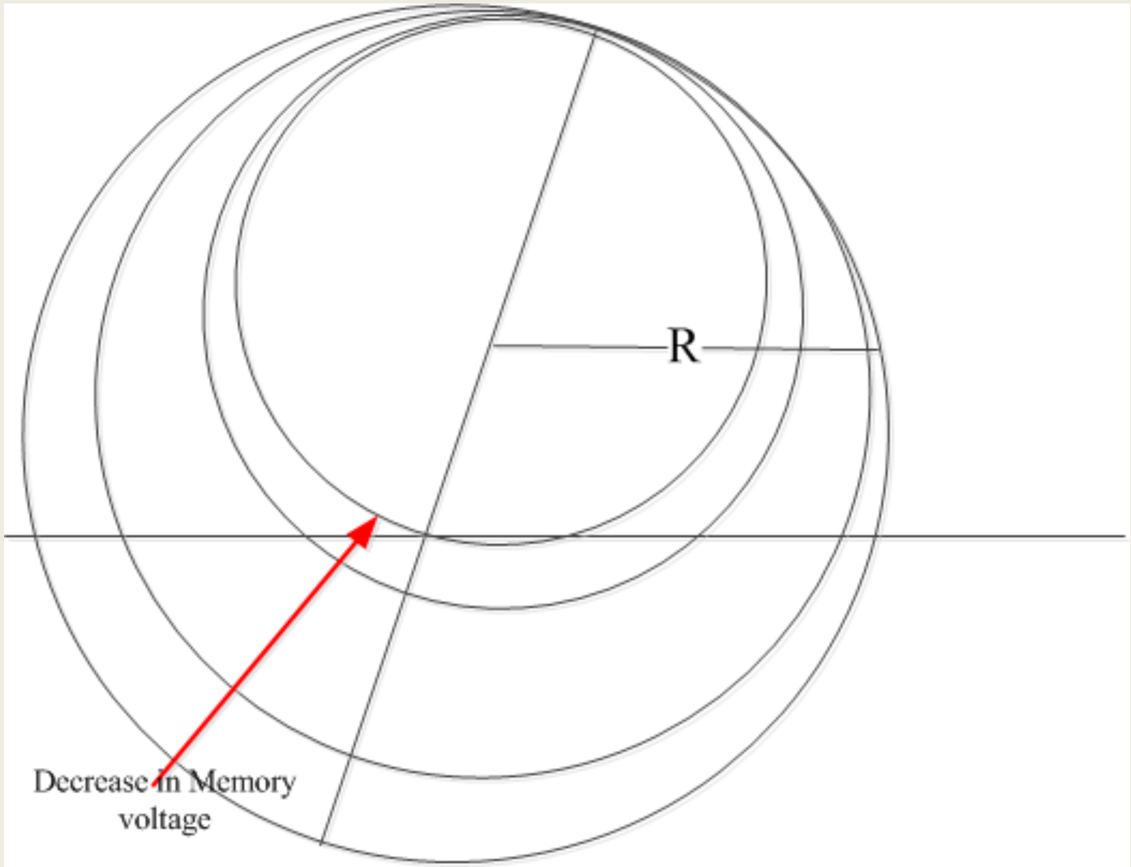






- Memory voltage allows detection of close-in faults when V=~0

Dynamic Mho Characteristics

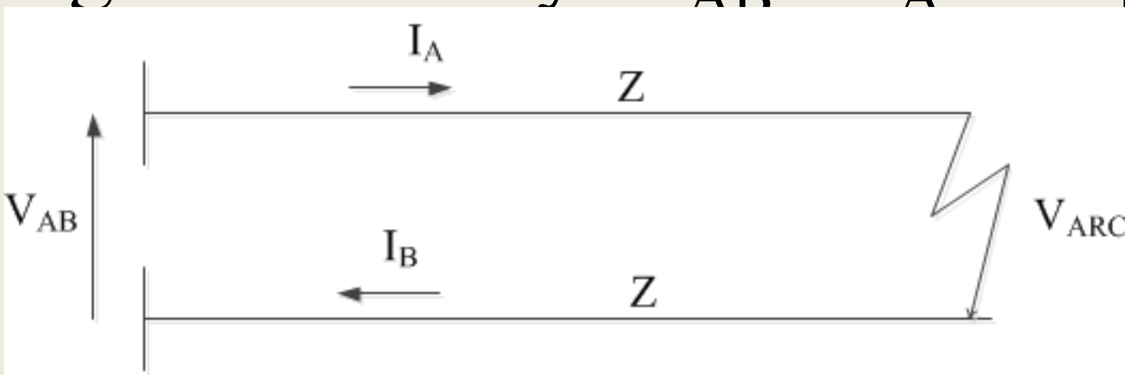


Mho Distance Element

Mho Distance Element	Fault Voltage, V	Fault Current, I	Polarizing voltage (Pre-fault or positive sequence memory)
A-G	V_A	$I_A+K_0I_N$	V_{AP}
B-G	V_B	$I_B+K_0I_N$	V_{BP}
C-G	V_C	$I_C+K_0I_N$	V_{CP}
A-B	V_{AB}	I_A-I_B	V_{ABP}
B-C	V_{BC}	I_B-I_C	V_{BCP}
C-A	V_{CA}	I_C-I_A	V_{CAP}

Resistance Coverage

- Phase to Phase faults:
- Voltage at the relay: $V_{AB} = I_A Z - I_B Z + V_{ARC}$

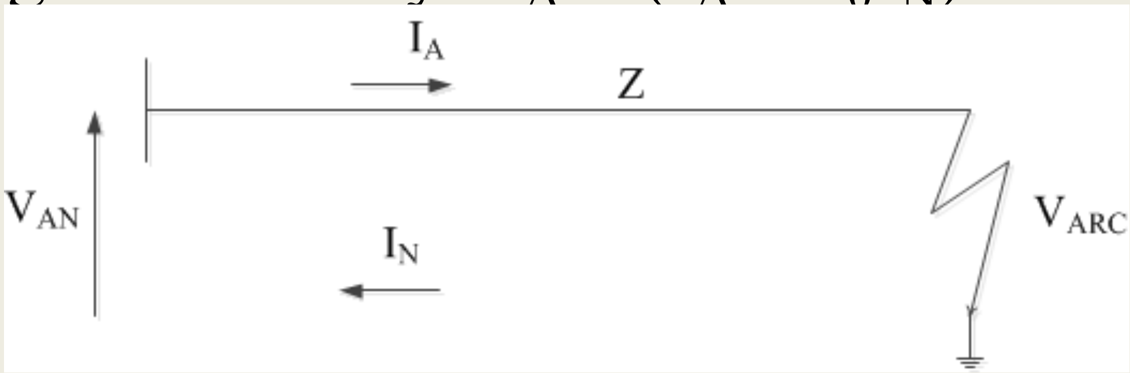


- $Z_{Calculated} = V_{AB}/(I_A - I_B) = Z + V_{ARC}/(I_A - I_B)$
- $= Z + V_{ARC}/(2*I) = Z + R_{ARC}/2;$

where $R_{ARC} = V_{ARC}/I_A$

Resistance Coverage

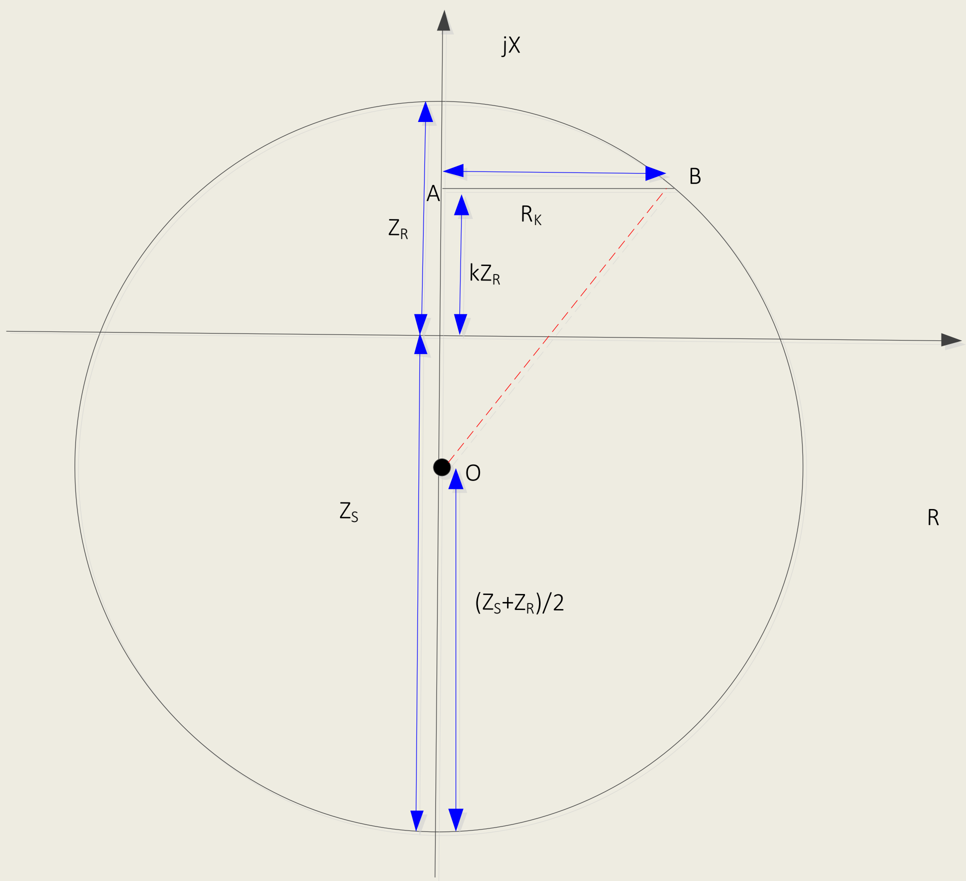
- Phase to Ground Faults
- Voltage at the relay: $V_A = (I_A + K_0 I_N) Z + V_{ARC}$

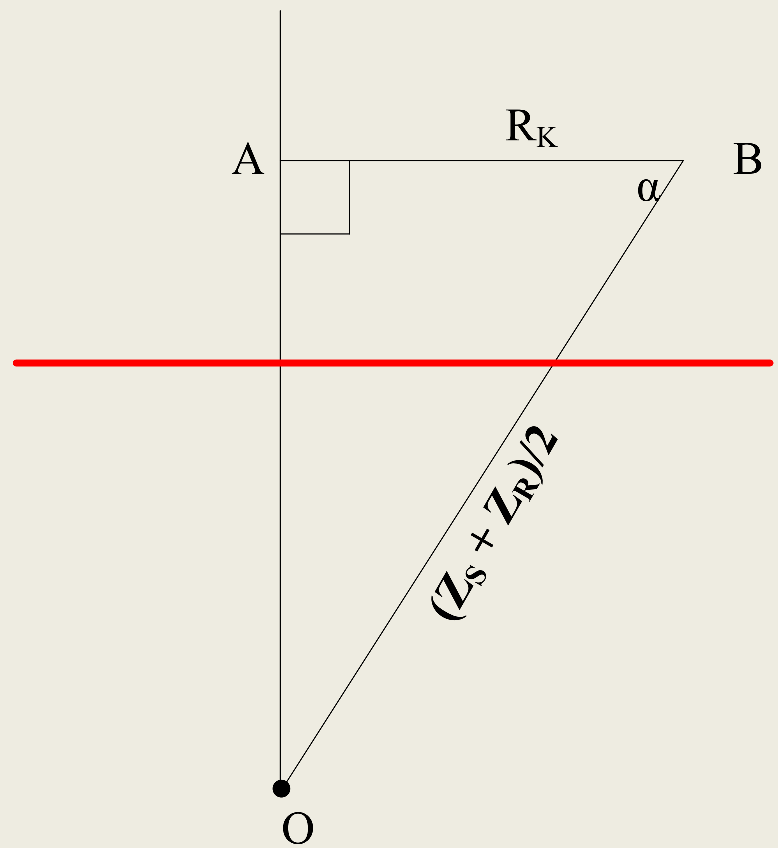


- $Z_{Measured} = V_A/(I_A + K_0 I_N) + R^* I_A/(I_A + K_0 I_N)$
- For a radially fed fault, $I_A = I_N$
- $Z_{calculated} = Z + R/(1+K_0)$
- Where, R = Arc resistance + parallel combination of Tower footing resistance and ground wire return path.

Resistance Coverage
Memory Polarized Mho Element

O- Center of the Circle; OA = $(Z_S + Z_R)/2 - (1\text{-}K)Z_R = (Z_S - Z_R)/2 + KZ_R$;
Radius, OB = $(Z_S + Z_R)/2$

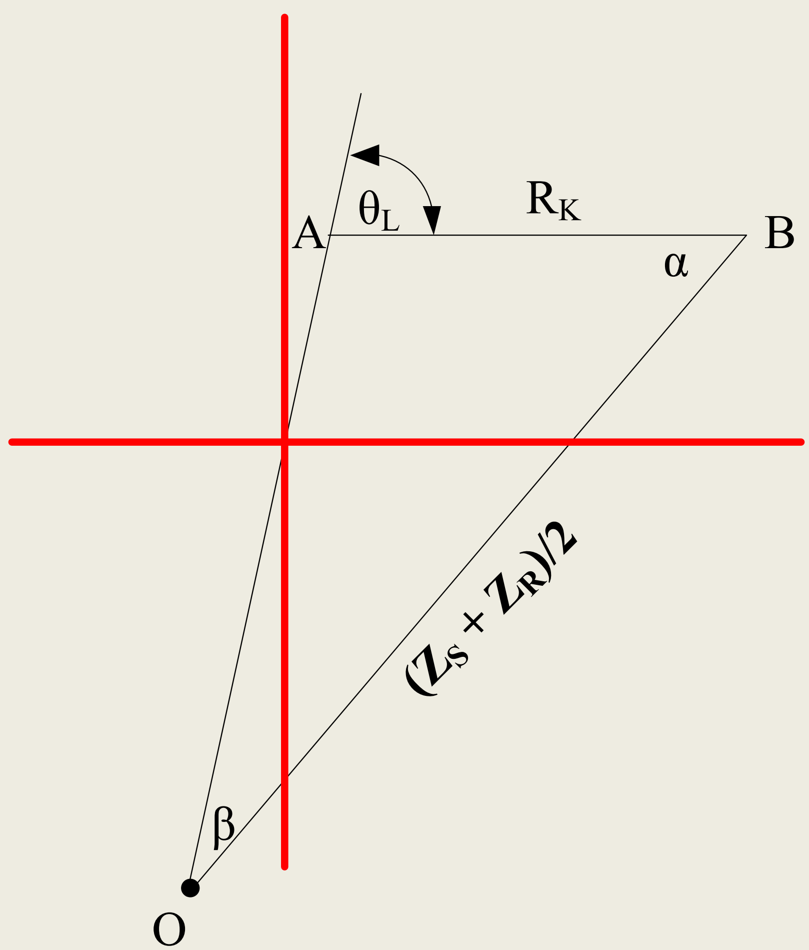




Resistance Coverage At location K

- $R_K = \sqrt{[((Z_S + Z_R)/2)^2 - ((Z_S - Z_R)/2 + KZ_R)^2]}$;
- Using $(A^2 – B^2) = (A-B)(A+B)$;
- $R_K = \sqrt{(Z_S + KZ_R)(1-K)Z_R}$
- Defining Z_S/Z_R as SIR (assuming the relay reach=Z_L)
- $R_K = Z_R\sqrt{(SIR+K)(1-K)}$
- For a fault at relay location, K=0
- $R_{close-in} = Z_R\sqrt{SIR}$
- $Z_R = m.Z_L$; m=0.85 for zone 1 and 1.25 for Zone 2
- $R_{close-in} = m.Z_L\sqrt{(Z_S/mZ_L)}$
- $R_{close-in} = Z_L\sqrt{(m*SIR)}$

Effect of considering the resistance of source and line



OB = Radius of the circle, $(Z_S+Z_R)/2$;
AB= resistance coverage, R_K, to be determined
OA = $(Z_S + Z_R)/2 - (1\text{-}K)Z_R = 0.5(Z_S\text{-}Z_R) + KZ_R$
Angle OAB = $(180\text{-}\theta_L)$ where θ_L is the line angle in degrees.

Using the law of sines:
$(OA/\sin\alpha) = (OB/\sin OAB) = R_K/\sin\beta$
Angle α is determined from
$\sin\alpha = (OA/OB)\sin(OAB) = (OA/OB)*\sin(180\text{-}\theta_L)$
$= (OA/OB)*\sin\theta_L = \frac{0.5(Z_S - Z_R) + KZ_R}{0.5(Z_S + Z_R)}\sin\theta_L$

Resistance Coverage –At location K

- $R_K = \frac{0.5(Z_S+Z_R)}{\text{Sin}\,\theta_L}\sin(\theta_L - \alpha)$
- $R_K = \frac{0.5(Z_S+Z_R)}{\sin\theta_L}\sin[\theta_L - \sin^{-1}\left(\frac{0.5(Z_S-Z_R)+KZ_R}{0.5(Z_S+Z_R)}\ \sin\theta_L\right)]$
- If the relay is set to $Z_{L,}$ Z_S=SIR*Z_L
- $R_K = [\frac{0.5(\text{SIR}+1)Z_L)}{\sin\theta_L}]\sin[\theta_L - \sin^{-1}\left(\frac{0.5(\text{SIR}-1)Z_L+KZ_L}{0.5(\text{SIR}+1)Z_L}\ \sin\theta_L\right)]$

Resistance coverage -R_K

- For $Z_R = m.Z_L$

- $\mathbf{R_K = [\frac{0.5(SIR+m)Z_L)}{\sin\theta_L}]\sin[\theta_L - sin^{-1}\left(\frac{0.5(SIR-m)Z_L + K*mZ_L}{0.5(SIR+m)Z_L}\ \sin\theta_L\right)}$
- $\mathbf{R_K = [\frac{0.5(SIR+m)Z_L)}{\sin\theta_L}]\sin[\theta_L - sin^{-1}\left(\frac{0.5(SIR-m) + K*m}{0.5(SIR+m)}\ \sin\theta_L\right)}$

- For SIR=0 and Relay set to cover 100% (m=1)
- $R_K = \frac{0.5Z_L}{sin\theta_L}$ $\text{Sin}[\theta_L - sin^{-1}(-1 * sin\theta_L)] = 0.5Z_L$ $\text{Sin}(2\theta_L)/\text{Sin}\theta_L$
- $R_K = Z_L \text{Cos}\ \theta_L$

Resistance coverage for close-in faults at various SIR values

Line with Z_L ohms at 80 degrees considered.

SIR	Resistance Coverage at location K (m=1) in PU (Z_L=1.0 PU)			
	K=0	K=0.25	K=0.5	K=0.75
0	0.174	0.479	0.5	0.393
1	1.0	0.926	0.784	0.544
5	1.916	1.632	1.28	0.813
10	2.476	2.068	1.582	0.964
15	7.849	7.842	7.836	7.829
20	10.293	10.287	10.280	10.273

Conclusions

- Voltage Drop across the arc either reduces with increase in current or remains constant for constant arc length
- Memory polarized Mho elements resistance coverage depends on the Source impedance and the type of fault.
- The Arc coverage with time depends on the memory voltage- whether it is held constant or whether it decreases with time.
- Resistance coverage at any point within the characteristics can be determined knowing the SIR.
- Modern fault program developers are planning to provide a macro/script to plot arcing faults using either Warrington/Westinghouse/Terzija's arc resistance equation.

Questions?